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Annabell Leigh Ulary

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ANALYSIS OF MODEL UNCERTAINTY IN HYDRAULIC MODELING: THE BSTEM APPLICATION TO THE OSAGE RIVER

by

ANNABELL LEIGH ULARY

A THESIS

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Approved by

Cesar Mendoza, Ph.D., Advisor
Charles D. Morris, Ph.D., P.E.
Jeffrey D. Cawlfield, Ph.D., P.E.

ABSTRACT

Uncertainty is inevitable when creating any kind of model. A model can be used in the most accurate way possible, if the uncertainties are understood. This study determines the level of uncertainty in the Bank Stability and Toe Erosion Model (BSTEM) of the Osage River downstream of Bagnell Dam between Lake Ozark, MO and Jefferson City, MO.

The statistical analysis of the BSTEM model was performed using the aid of SAS statistical computer modeling software. There were 4 different analysis values used to determine the best fit model for all dependent variables. These values include the F-test, the coefficient of determination, mean squared error, and Mallow's C_p. The F-test is used to determine that there is indeed a relationship between the independent variables and the dependent variables, whereas the other 3 values help narrow down the simplified statistical models to determine the best fit model for each dependent variable.

There were 4 different BSTEM outputs that were used in the uncertainty analysis. These 4 dependent variables are average applied boundary shear stress, factor of safety, maximum lateral retreat and eroded area – total. The statistical analysis determined how many best fit statistical models each variable appeared in and this information helped to determine the variables affecting the BSTEM model. The variables that appeared in all the best fit statistical models had a large impact on the BSTEM model, whereas the ones that did not show up in a best fit statistical model had a small effect on the BSTEM model. The factor of safety analysis yielded results that were inconclusive, while the other three variables had a confidence level ranging from 76.7% up to 90.6%, with an average confidence of over 80% for the entire BSTEM model.

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TABLE OF CONTENTS

SECTION

APPENDICIES

LIST OF TABLES

1. INTRODUCTION

1.1. ASPECTS OF UNCERTAINTY

When using or creating a hydraulic model the degree of certainty should be something that is taken into account. There are several different parts of modeling that can lead to uncertainty: variable uncertainty, model uncertainty and parametric uncertainty. Variable uncertainty comes from the data entered into the model; due to measurement error or by approximating values. Model uncertainty is the uncertainty that arises from the computations within the model itself. Parametric uncertainty is caused by rounding errors [2].

This particular study focuses on the model uncertainty of the Bank Stability and Toe Erosion Model (BSTEM) hydraulic model. When looking for just the model uncertainty the variable and parametric uncertainties are assumed to be negligible. In other words all the data that is being put into and taken from the model is assumed to be as accurate as the model allows.

1.2. BANK STABILITY AND TOE EROSION MODEL

BSTEM is a model developed by the Department of Agriculture to determine erosion and bank stability in a stream. BSTEM uses the bank geometry, geotechnical data and the hydraulic conditions to determine the effects of erosion on a stream. The user is able to input the cross-section geometry, and up to 5 different soil layers. In addition, channel parameters such as channel length, slope and any vegetation that can be found along the bank are input into the model. The water surface elevation, depth to the phreatic surface, and duration of the current flow conditions are added into the model. From the data that was entered into BSTEM the “Bank-Stability Model” section of the model computes the factor of safety (FS) of the cross-section. This is directly related to the stability of the bank at that location. Similarly the “Toe-Erosion Model” computes the total amounts of erosion for the given soil parameters and flow conditions.

1.3. DATA

The channel that was used to analyze BSTEM was the Osage River, which is located downstream of Bagnell Dam and Lake of the Ozarks. A previous study by Heinley [1] provided geometric profiles along with soil types for 10 profiles along the river. Given these soil types, the values for the friction angle, cohesion coefficient, saturated unit weight, and the unsaturated strength parameter can be determined. The foliage coverage is considered to be zero, so all soils would be exposed to the channel flow. The phreatic surface was assumed to be at the top of the bank for all cross-sections as this is the most unstable condition. Bagnell Dam outflow hydrograph, from Ameren's website, along with HEC-RAS helped to determine a flow elevation in each cross-section [4]. From the hydrograph produced by HEC-RAS the average plus one standard deviation on either side of the flow elevations were used in the uncertainty analysis. Flow duration values of 0.5, 1, 2 and 5 hours was used. Each cross-section was run 4 different times with a combination of flow elevation and duration that were chosen arbitrarily [1]. The initial data that was input into BSTEM along with the results from each run can be found in Appendix A.

1.4. PURPOSE AND SCOPE

The purpose of this thesis is to determine the degree of uncertainty for the hydraulic model BSTEM. The uncertainty will not only be quantified on an overall scale, but the different variables will also be ranked according to their influence that they have on the model output. All calculations made using the data collected for the Osage River, downstream of Bagnell Dam.

The scope of this thesis includes a review of literature pertaining to uncertainty in hydrologic and hydraulic modeling, the collection and analysis of data, an overall degree of uncertainty for BSTEM; and a ranking of the variable's influence on the model output.

1.5. THESIS ORGANIZATION

This thesis is comprised of five main sections. Section 1 is the introduction to the thesis. Section 2 contains a review of literature discussing uncertainty in hydrologic and hydraulic modeling. Section 3 discusses the statistical modeling that was used in the analysis of the data collected from BSTEM. Section 4 shows and discusses the results that were obtained from the statistics model. The final section, Section 5, states the conclusions drawn from the analysis performed and provides recommendations for future uses of BSTEM.

2. REVIEW OF LITERATURE

2.1. GENERAL

Uncertainty is present in all hydraulic models in several different ways. There can be uncertainties from the variables that are put into the model, errors in the model itself and errors that are due to rounding errors. Each of these are considered when creating a model [2]. It is no longer acceptable to state that there is uncertainty, without any determination of the source or the amount of uncertainty that exist. Clients and project managers should be fully informed of the amount and source of uncertainty so that they have the ability to make sound decisions concerning the model [5].

2.2. VARIABLE UNCERTAINTY

The variable uncertainty can be divided into two different subsections: 1) the input data and 2) calibration. It is important to distinguish between these two types of variable uncertainty so that corrections or adjustments can be made to the model to make the models as accurate as possible [5].

2.2.1. Input Data Uncertainty. The uncertainty in the data input into a model can stem from measurement errors or from uncertainty in the equations that are used to determine the variable. The measurement errors are starting to lessen due to advances in technology in taking accurate measurements. However, there are still some hydrologic measurements that still have a high degree of inaccuracy (e.g. velocities in a natural channel, precipitation data, etc.). The majority of the time these errors are not accounted for when determining the uncertainty of a model because they are hard to quantify [2]. The types of input data uncertainty that can be quantified comes from running another model or equation. Variables such as the roughness coefficient are calculated using another equation. Variables from other calculations have a quantifiable uncertainty and professionals, such as Warmink [6], have done analysis to determine errors in variables from previous calculations. Using an analysis similar to Warmink’s, allow a modeler to pinpoint the source of the uncertainty and make adjustments if necessary [6].

2.2.2. Calibration Uncertainty. There are several models that use curves (e.g. pipe roughness and rating curves) in their calculations [2]. The majority of the curves used in hydraulics are created using best fit lines and so the uncertainty will follow along the certainty of the curve and the way that the curve was created. If a curve is created using a bunch of data points the uncertainty of the curve will be lowered. If the curve is created off a handful of data points the degree of uncertainty will be high. It is important to determine the level of certainty of the calibration material to find the best calibration data possible in order to help eliminate sources of high uncertainty.

2.3. MODEL UNCERTAINTY

The term model uncertainty lumps together a few different sources of uncertainty that have to relate to the model. These different sources include over simplification of the model and the design limits of the model [5].

2.3.1. Over Simplification of the Model. In modeling there are judgment calls that must be made in constructing the model. If a model is too complex there are more calculation and rounding errors, which are hard to quantify. However, if a model is over simplified then elements that might greatly influence the overall results could be excluded. An example of this is in the calculations hydrologic process (e.g. infiltration and evapotranspitation). All models don’t use the same variables and they don’t always use the same constants. When determining a model to use, it is best to find the one with the greatest confidence, no matter how complex or simple the model might be.

2.3.2. Design Limits of the Model. When a model is created, there is a specific range of conditions where the model is most accurate and conditions when the model might have more uncertainty. For example, the statistical models that were created by this study are specific to the Osage River downstream of the Bagnell Dam; the findings from this model might not be accurate in locations where the vegetation is different or when the soil layers are in a different order. It is important to know and understand the limits of a specific model. If the data is out of the limits of the model there is a large increase in model uncertainty. This uncertainty can be quantified with a statistical analysis. Once this analysis has been performed it is important to make the necessary adjustments to the model to best fit the conditions of the data [5].

2.4. PARAMETRIC ERROR

Parametric error is caused by the rounding of numbers. Parametric error also includes imperfect processes in the modeling due to the lack of understanding the interaction of these factors [2]. An example of a rounding error would be the calculation of the area or circumference of a circle. The equations for these values involve π, which is rounded off at the hundredths. However, when calculating the area for a rather large area, rounding off the value might produce an erroneous result.

The lack of knowledge on the processes involved in a model could cause errors in data included in the model. If all the correct variables are not included in the model, then the results may be skewed. This is one of the parameters that are important in a statistical analysis. When determining the best fit model, if all the correct parameters are not included, then the model will be under defined and not fit the data accurately.

3. STATISTICAL MODEL ANALYSIS

3.1. MULTIPLE LINEAR REGRESSION

Multiple linear regression is a statistical model where several independent variables act on a dependent variable [3]. In the case of BSTEM, all the input data is considered as independent variables. These independent variables act on the dependent variables, which is the program output data. For this thesis it has been determined that there are 36 different independent variables with 4 main dependent variables. The 36 independent variables and the 4 dependent variables can be found in Table 3.1 below. Abbreviations for each variable are also displayed in the third column; these will be helpful when looking at that analysis of the data. The “Notes” column on the table explains where some of the variables come from or what parts of the abbreviations mean.

Table 3.1. Variables Used in Analysis

Type	Name (units)	Abbreviation	Notes
Independent	Reach Length (m)	reachL	For cross-section being analyzed. Determined from field data.
Independent	Reach Slope (m/m)	reachS	For cross-section being analyzed. Determined from field data.
Independent	Flow Elevation (m)	FlowELE	For cross-section being analyzed. Determined from HEC-RAS.
Independent	Flow Duration (hrs)	TFlow	For cross-section being analyzed. Chosen arbitrarily at 0.5, 1, 2 or 5 hours
Independent	Critical Shear *(For the Toe)* (Pa)	CritShr	The same values are used for all layers. Determined from soil type
Independent	Erosion Coefficient *(For the Toe)* (cm^3/Ns)	EroCoeff	The same values are used for all layers. Determined from soil type
Independent	Thickness *(For all layers)* (m)	Thck#	There will be one of these variables for each layer. Determined from field data.
Independent	Wetted Perimeter *(For all layers)* (m)	WetP#	There will be one of these variables for each layer. Determined from field data.
Independent	Friction Angle *(For all layers)* (degrees)	FriAng#	There will be one of these variables for each layer. Determined from soil type.
Independent	Cohesion Coefficient *(For all layers)* (kPa)	Coh#	There will be one of these variables for each layer. Determined from soil type.
Independent	Saturated Unit Weight *(For all layers)* (kN/m^3)	SUW#	There will be one of these variables for each layer. Determined from soil type.
Independent	Unsaturated Strength Parameter *(For all layers)* (degrees)	Unsat#	There will be one of these variables for each layer. Determined from soil type.
Dependent	Average Applied Boundary Shear Stress (Pa)	ABSS	Determined from BSTEM output
Dependent	Factor of Safety	FS	Determined from BSTEM output
Dependent	Maximum Lateral Retreat (cm)	MLR	Determined from BSTEM output
Dependent	Eroded Area – Total (m^2)	Total	The total amount of erosion that occurred

3.2. STATISTICAL MODEL

For a multiple linear regression statistical model it is assumed that the depended variable *y* can be explained by the independent, or predictor variables x_i, using

$$y = \beta_0 + \beta_1 x_{i,1} + \cdots + \beta_{p-1} x_{i,p-1} + \varepsilon_1 \qquad (1)$$

Where there are *p-1* predictor variables, β_i are fixed unknown variables and ε represents the random error that occurs. While the *x* variables are referred to as predictor variables, the model that is being created is an explanatory statistical model meaning the relationship between *y* and the variables are being explained, rather than trying to predict future *y* values.

The way that the value of β_i and ε are found is a method called least squares. The least squares method minimizes the value of equation 2.

$$\sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_{\,i,1} - \beta_{p-1} x_{i,p-1})^2 \qquad (2)$$

The computer program SAS will help select the variables that are included in the statistical models. SAS uses a maximum R^2 improvement method to determine the β_i values as well as the independent variables that are included in the particular statistical model. This method of determination analyzes is the most combination of variables because it examines all the possible variable combinations for a statistical model before moving on to the next variable level. For example SAS will analyze all the possible 4 variable combinations and determine the variable combination that produces the largest R^2 value. Once the best 4 variable model is determined SAS will analyze all the 5 variable combinations and repeat the process until R^2 has reached its highest possible value. The statistical models use a reduced amount of variables in order to determine which variables impact the results the most. The meaning of R^2 will be explained in the next section [3].

3.3. ANALYSIS METHOD

There are four different elements that need to be examined in order to accurately determine the best statistical model for each dependent variable. First it needs to be verified that there is indeed a relationship between each dependent variable and the set of independent variables. Once this relationship has been verified each statistical model can be examined to determine the best fit statistical model for each dependent variable. The three different values that help rank the fit of the different statistical models are: 1) coefficient of determination 2) mean squared error and 3) Mallow C_p [3].

3.3.1. Verifying the Relationships. In order to verify that there is in fact a relationship between the dependent and independent variables the following hypothesis must be tested

$$H_0 : \beta_1 = \cdots = \beta_{36} = 0$$

versus

$$H_A : \beta_i \neq 0 \; for \; atleast \; one \; i = 1, 2, \ldots, 36$$

The H_0 hypothesis states that there is no linear relationship between any of the independent variables and the dependent variable being analyzed. Whereas the H_A hypothesis states that at least one independent variable has a significant relationship with the dependent variable. This hypothesis is tested using the F-test and the statistical models found to have significant F values are used. The F value is calculated by

$$F = \frac{MSR}{MSE} \qquad (3)$$

Where MSR is the mean squared regression and MSE is the mean squared error. The calculated F value is compared to the values in the F tables for a specific degree of confidence (α). If the calculated ones are larger than the ones that are listed then it would be considered significant [3]. The SAS computer program will automatically determine the highest degree of confidence that the F value would be considered significant.

3.3.2. Coefficient of Determination. The coefficient of determination, also known as R^2, measures how much of the variability of y is explained by the x variables. This is found using the sum of squares regression (SSR) and the sum of squares total (SST) in the following formula.

$$R^2 = \frac{SSR}{SST} \tag{4}$$

The closer the R^2 value is to 1 the better predictability the statistical model. However just because a statistical model has a high R^2 value doesn't mean that it is the best fit statistical model as there are other factors to examine [3].

3.3.3. Mean Squared Error. The mean squared error (MSE) is an estimation of the variance for a particular statistical model. The variance explains how widely spread the statistical model data is from the actual data. A statistical model that best represents the actual data it is desired to have a statistical model with a low MSE [3].

3.3.4. Mallow's C_p. Mallow's C_p is an estimation of the difference between a reduced statistical model, the model selected by SAS, and the actual model, the data given from BSTEM. These two models are compared using the sum squares error of the reduced model, SSE_p, and the mean square error of the actual model, MSE_{full}. The equation for C_p is

$$C_p = \frac{SSE_p}{MSE_{full}} - n + 2(p+1) \tag{5}$$

Where n is the number of observations that are used and p is the number of variables in the reduced statistical model. When evaluating based on the C_p statistic there is a preference based on smaller statistical models where $C_p \leq p + 1$ and the closer to $p+1$ the better. The reason for this is because when $C_p > p + 1$ the statistical model is under defined and when $C_p = p + 1$ the statistical model is using all the variables that are in the actual model [3]

4. RESULTS AND DISCUSSION

4.1. GENERAL

Once all the results had been put into the tables that can be found in the appendixes, it was clear that the critical shear and the erosion coefficient for the toe do not appear to affect any of the statistical models that were analyzed. This can be explained by the fact that for the analyzed section of the Osage River location these values did not change.

4.2. AVERAGE APPLIED BOUNDARY SHEAR STRESS

For the output variable of the average applied boundary shear stress there were 34 different statistical models analyzed, using 20 different variables. A sample of the results can be found in Table 4.1. Appendix B provides the full analysis results for the average applied boundary shear stress. The "p" column from the table, lists how many variables are in a specific statistical model. The F value is the one that was calculated using the data from that particular statistical model. The "α" column shows the degree of certainty that the H_0 hypothesis is rejected. For the average applied boundary shear stress variable, the H_0 hypothesis is rejected, with a degree of confidence less than 0.0001 in all examined statistical models. This means in all these statistical models the F value is significant and that there is indeed a correlation between at least one independent variable and the average applied boundary shear stress.

Statistical models 32, 33 and 34 had the same high R^2 value of 0.9055. From this point on these will be the statistical models that will be used for the analysis. Of these 3 statistical models the MSE values for statistical models 32 and 33 were so close that both statistical models would be considered for the next analysis. By looking at the C_p value it can be determined that statistical model number 32 is the most accurate statistical model to determine the average applied boundary shear stress, since it is the statistical model with the C_p value that is closer to p+1. The 16 different variables that are used in statistical model 32 can be found in Table 4.2, along with β_i values that are associated with each variable and the intercept value, β_0 needed to align the model with the actual data when using equation 1.

Table 4.1. Sample of the Average Applied Boundary Shear Stress Results

Model #	p	F Value	α	R^2	MSE	Cp
1	1	29.63	<0.0001	0.4256	30872.000	107.894
2	2	31.50	<0.0001	0.6177	21075.000	61.105
3	3	24.80	<0.0001	0.6619	19125.000	51.863
4	4	20.49	<0.0001	0.6890	18071.000	46.996
5	5	18.64	<0.0001	0.7213	16640.000	40.772
11	6	29.57	<0.0001	0.8352	10119.000	13.845
12	7	29.75	<0.0001	0.8597	8872.963	9.642
27	14	18.37	<0.0001	0.9050	7563.890	12.128
30	15	16.54	<0.0001	0.9051	7844.627	14.097
31	16	14.91	<0.0001	0.9051	8157.985	16.096
32	16	14.97	<0.0001	0.9055	8129.315	16.011
33	16	14.97	<0.0001	0.9055	8125.568	16.000
34	17	13.53	<0.0001	0.9055	8464.110	18.000

Table 4.2. Variables in ABSS Statistical Model 32

x_i	β_i	x_i	β_i	x_i	β_i
Intercept	-867236	WetP1	2896.434	FriAng3	14443
reachL	12.032	thck2	51144	Thck4	-44200
reachs	13256589	WetP2	-208.844	WetP4	-422.038
FlowELE	111.553	Coh2	-1103.969	Coh4	16013
Tflow	11.919	Thck3	14677	WetP5	-421.664
Thck1	17.764	WetP3	-683.269		

4.3. FACTOR OF SAFETY

The statistical analysis results for the Factor of Safety variable analyzed 23 models, with a range of 1 to 17 different variables. A sample of the results can be found in Table 4.3. Refer to Appendix C for the full results refer to Appendix C. The F value is considered to be significant when using an α value of less than 0.0001 and greater than 99.9999% certainty and in turn H_0 would be rejected. However the F value determined for over half of the statistical models appear to be strange. It is not very often that the F value is equal to infinity. According to the coefficient of determination there were 10 different statistical models that produced a completely accurate statistical model. When looking at the mean squared error there are 9 variables that are considered to be completely accurate, this is evident by the 0 values for the MSE variable. The Mallow's C_p statistic is completely inconclusive, as all the values are the same and none of them are close to $p+1$, which would be the ideal value. Based on the results from the 4 tests it can be concluded that the results for the Factor of Safety variable are inconclusive.

Table 4.3. Sample of Factor of Safety Results

Model #	p	F Value	α	R^2	MSE	Cp
1	1	14.19	0.0005	0.2619	1.83E+15	0.000
2	2	20.36	<0.0001	0.5108	1.24E+15	0.000
3	3	22.96	<0.0001	0.6444	9.27E+14	0.000
4	4	31.97	<0.0001	0.7756	6.01E+14	0.000
5	5	49.58	<0.0001	0.8732	3.49E+14	0.000
6	6	112.51	<0.0001	0.9507	1.39E+14	0.000
9	7	249.46	<0.0001	0.9809	5.56E+13	0.000
13	9	18589.9	<0.0001	0.9998	5.92E+11	0.000
14	9	1.4E+08	<0.0001	1.0000	7.98E+07	0.000
15	9	∞	<0.0001	1.0000	0.00E+00	0.000
17	11	∞	<0.0001	1.0000	0.00E+00	0.000
21	15	∞	<0.0001	1.0000	0.00E+00	0.000
22	16	∞	<0.0001	1.0000	0.00E+00	0.000
23	17	∞	<0.0001	1.0000	0.00E+00	0.000

4.4. MAXIMUM LATERAL RETREAT

There were 32 different statistical models analyzed for the maximum lateral retreat. The full results can be found in Appendix D; Table 4.4 provides small sample of those same results. Once again the F values proved to be above the values that can be found in the F-tables, as shown by the α column. The H_0 hypothesis is rejected with over

99.99% confidence for all statistical models. The R^2 value narrows down the results to 6 different statistical models. Of these 6 statistical models the MSE analysis narrows down the statistical models again to statistical models 27 and 28 because they have the same small MSE value. The Mallow's C_p value for both of these statistical models is the same; they are also the same size which means that either statistical model would be accurate to describe the maximum lateral retreat for this river location. The difference between the 2 statistical models is that statistical model 27 includes the cohesion coefficient for layer 2 and statistical model 28 includes the slope of the reach. Since the cohesion coefficient for layer 2 and the slope of the reach, can be changed out without much of a chance in the rest of the equation, it can be concluded that they have the same effect on the maximum lateral retreat results.

Table 4.4. Sample of Maximum Lateral Retreat Results

Model #	p	F Value	α	R^2	MSE	Cp
1	1	13.05	0.0008	0.2460	41503.000	39.586
2	2	16.56	<0.0001	0.4593	30524.000	19.635
3	3	14.46	<0.0001	0.5330	27059.000	14.055
6	4	18.13	<0.0001	0.6622	20103.000	2.762
10	6	14.21	<0.0001	0.7089	18310.000	1.951
12	7	12.16	<0.0001	0.7146	18479.000	3.363
22	13	6.86	<0.0001	0.7610	18792.000	10.591
23	14	6.14	<0.0001	0.7611	19483.000	12.585
26	15	5.70	<0.0001	0.7667	19756.000	14.006
27	16	5.14	0.0001	0.7668	20541.000	16.000
28	16	5.14	0.0001	0.7668	20541.000	16.000
29	17	4.64	0.0003	0.7668	21397.000	18.000
30	17	4.64	0.0003	0.7668	21397.000	18.000
31	17	4.64	0.0003	0.7668	21397.000	18.000
32	17	4.64	0.0003	0.7668	21397.000	18.000

Table 4.5. Variables used in MLR Statistical Models 27 and 28

	x_i	β_i	x_i	β_i	x_i	β_i
Model 27	Intercept	766264	FricAng1	-23486	WetP3	1100.152
	reachL	13.403	Coh1	-25939	Coh3	12476
	Flow ELE	-1.0752	Thck2	-59830	WetP4	255.843
	Tflow	-2.638	WetP2	6042.540	Coh4	-17640
	Thck1	91.533	Coh2	-951.775	WetP5	256.600
	WetP1	-1318.179	Thck3	3920.580		
Model 28	Intercept	637265	WetP1	-1785.390	WetP3	1102.384
	reachL	13.467	FricAng1	-20720	Coh3	7412.022
	reachS	2339774	Coh1	-22588	WetP4	257.335
	Flow ELE	-1.109	Thck2	-52293	Coh4	-13459
	Tflow	-2.639	WetP2	5862.915	WetP5	255.077
	Thck1	92.456	Thck3	-807.278		

4.5. ERODED AREA – TOTAL

The full results for the eroded area – total analysis can be found in Appendix E; Table 4.6 displays a sample of these results. There were 26 different statistical models analyzed for the eroded area – total analysis and the statistical models had anywhere from 1 to 17 different variables. The F value had more variability in this analysis compared to the others. The smaller statistical models in this analysis reject the H_0 hypothesis at α values higher than what have been seen in the other models. However, for the larger statistical models the H_0 hypothesis is rejected at the confidence of more than 99.99%, which is the same as the values in the other models. The analysis is narrowed down to 3 statistical models using the R^2 values and selecting only the statistical models with the highest value. From narrowed down statistical models, model number 24 has the lowest MSE value making it the best fit statistical model. The Mallow's C_p analysis confirms that model 24 is the best fit statistical model for the eroded area – total. The specifics of model 24 can be found in Table 4.7, this includes the variables that are used along with the parameter estimates for each.

Table 4.6. Sample of Eroded Area – Total Results

Model #	p	F Value	α	R^2	MSE	Cp
1	1	6.24	0.0167	0.1349	125.050	73.252
2	2	5.10	0.1080	0.2074	117.508	65.929
4	3	4.81	0.0061	0.2754	110.260	59.189
5	4	7.26	0.0002	0.4397	87.556	40.053
6	5	7.16	<0.0001	0.4986	80.525	34.476
7	6	7.50	<0.0001	0.5626	72.262	28.252
8	7	10.12	<0.0001	0.6758	55.141	15.698
9	8	9.41	<0.0001	0.6951	53.416	15.206
20	13	6.79	<0.0001	0.7591	49.746	16.980
21	14	6.08	<0.0001	0.7592	51.568	18.968
22	15	5.47	<0.0001	0.7593	53.533	20.957
23	15	7.53	<0.0001	0.8129	41.619	14.067
24	16	6.81	<0.0001	0.8134	43.163	16.000
25	17	6.15	<0.0001	0.8134	44.961	18.000
26	17	6.15	<0.0001	0.8134	44.961	18.000

Table 4.7. Variables used in Total Statistical Model 24

x_i	β_i	x_i	β_i	x_i	β_i
Intercept	455146	WetP1	-1248.184	WetP3	-54.149
reachL	-2.712	Coh1	-1017.110	Coh3	-6574.232
reachS	-5091037	WetP2	576.733	WetP4	-41.414
Flow ELE	0.966	FriAng2	-7825.605	Thck5	5157.372
Tflow	0.156	Coh2	-6656.988	WetP5	-41.999
Thck1	-5.960	Thck3	-1634.386		

5. CONCLUSIONS AND RECOMMENDATIONS

5.1. SIGNIFICANT VARIABLES

The significant variables are determined for the full BSTEM model at this location. The variables that are considered significant are the ones that are involved in the best fit statistical models for each variable. The number of statistical models that each variable was used in can be found in Table 5.1. It is important to remember that for this analysis the factor of safety analysis was inconclusive and the maximum lateral retreat had 2 statistical models that were considered acceptable. The variables are ranked from "Very High Significance," when the variable is used in all 4 statistical models, to "Very Low Significance," when the variable appears in none of the best fit statistical models.

Table 5.1. Number of Statistical Models Each Variable was Used In

Variables used in 4 Models (Very High Significance)
Reach Length Flow Elevation Flow Duration Thickness (layers 1 and 3) Wetter Perimeter (all layers)
Variables used in 3 Models (High Significance)
Reach Slope Thickness (layer 2) Cohesion Coefficient (layers 1, 2, 3 and 4)
Variables used in 2 Models (Moderate Significance)
Friction Angle (Layer 1)
Variables used in 1 Model (Low Significance)
Friction Angle (layers 2 and 3) Thickness (layers 4 and 5)
Variables used in 0 Models (Very Low Significance)
Critical Shear (for the Toe) Erosion Coefficient (for the Toe) Saturated Unit Weight (all layers) Unsaturated Strength Parameter (all layers)

For the 10 cross sections, downstream of Bagnell Dam, it is important that the variables that fall under the “very high significance” section be carefully measured, since these are the variables that will have the largest impact on the overall results of the BSTEM model. However, that does not mean that the variables that did not have a large significance should be forgotten or left out, these variables might still have an impact in the overall all BSTEM model.

5.2. ACCURACY OF EACH MODEL

The degree of accuracy for each model ultimately depends on the accuracy of the data that is input into the model. However, if it is assumed that all the given data is fully accurate, it is important to understand the accuracy and the limitations of each model, or the confidence. The confidence for each dependent variable is the coefficient of determination that correlates with the model. The confidence for each dependent variable can be found in Table 5.2. For these confidence values the coefficient of determination is displayed as a percentage of confidence. The overall average confidence for BSTEM is greater than 80% for the Osage River downstream of Bagnell Dam.

Table 5.2. Percent of Confidence for Each Dependent Variable

Average Applied Boundary Shear Stress	90.55%
Factor of Safety	N/A
Maximum Lateral Retreat	76.68%
Eroded Area - Total	81.34%

5.3. LIMITATIONS AND ALTERNATIVE METHODS

There are several assumptions that have to be made with a multiple linear regression analysis. The main assumption is that the data will fit a linear regression. Several different analysis types and while linear regression is the most common, other analysis that could better fit the data. It is also assumed that the only errors made in the calculations and that all the data that is put into the program is correct. If one of the variables was measured incorrectly then the whole analysis should to be rerun because there is no accurate way to predict how the mistake would affect the analyzed statistical models. Another limitation is when it comes to ranking the variables one at a time on the effect that they have on the model. The only way to rank the variables using this method is by looking at the number of times that the variable shows up in the best fit statistical models.

Another analysis that could be done is a quartile regression. With the quartile regression the best fit model would fall into a specific quartile range, whereas with the multiple linear regression the best fit statistical model falls closer to the median. In addition instead of performing a least squares analysis, a least absolute deviation analysis can be performed. This is similar to the least squares method but rather than squaring the errors and minimizing them, the absolute value of the deviation is taken and minimized. There are generally the same assumptions made with this method, but it is not as commonly used.

5.4. RECOMMENDATIONS FOR FURTHER ANALYSIS

The analysis that has been performed on BSTEM is specific for the Osage River downstream from Bagnell Dam, and locations with similar physical properties, including the soil conditions for each layer. If BSTEM is being used for a location that varies from the conditions that are found in this section of the Osage River, it is recommended that another statistical analysis of the data be performed. Also if the material in a specific layer is transposed, it is important to compare layers of like materials to one another.

There were also specific independent variables that either had little to no varying data (e.g. vegetation, erosion coefficient). If there is a location where there is changing data for these variables it is extremely important to compute another statistical analysis because the overall confidence may vary slightly. With the addition of different bank vegetation information there is also the likely possibility that the factor of safety would yield useful results. If this is ever the case, it is important to perform a statistical analysis on the factor of safety because the results from this test were inconclusive.

APPENDIX A.

INITIAL INUT DATA

Cross Section	*Trial*	Output from BSTEM				reachL	reachS	FlowELE	Tflow
		ABSS (Pa)	FS	MLR (cm)	Total (m^2)	(m)	(m/m)	(m)	(hrs)
CS1	1	149.13	99999999	0.000	0.0	7177.5	0.02271	165.0	1
	2	687.72	99999999	9.901	15.0	7177.5	0.02271	170.2	0.5
	3	435.54	99999999	0.024	0.0	7177.5	0.02271	167.6	5
	4	711.64	99999999	0.000	0.0	7177.5	0.02271	170.2	2
	5	144.85	99999999	0.000	0.0	7177.5	0.02271	165.0	5
CS3	1	348.05	99999999	251.810	12.5	11651.5	0.01390	167.2	2
	2	157.8	99999999	20.163	0.3	11651.5	0.01390	164.8	1
	3	445.71	99999999	20.163	0.0	11651.5	0.01390	169.6	0.5
	4	403.6	99999999	20.163	0.0	11651.5	0.01390	167.2	5
	5	445.92	99999999	20.164	0.0	11651.5	0.01390	169.6	1
CS4	1	414.91	0.4	1.199	7.0	8312	0.01967	166.7	2
	2	91.85	0	61.019	13.4	8312	0.01967	164.7	1
	3	835.41	0	460.807	2.5	8312	0.01967	168.8	5
	4	131.82	0	247.450	1.2	8312	0.01967	164.7	0.5
CS5	1	642.18	99999999	424.503	39.5	6871.5	0.02376	167.5	2
	2	190.36	99999999	422.090	2.0	6872.5	0.02376	165.1	1
	3	837.63	99999999	330.788	0.3	6873.5	0.02376	167.5	0.5
	4	274.57	99999999	353.920	1.0	6874.5	0.02376	165.1	5
CS6	1	221.62	99999999	502.105	59.0	14146	0.01122	163.8	0.5
	2	135.8	99999999	550.962	27.4	14147	0.01122	161.1	1
	3	306.4	99999999	702.087	4.3	14148	0.01122	163.8	5
	4	461.48	99999999	635.359	0.1	14149	0.01122	166.5	2
CS8	1	23.37	0	0.250	2.0	17871.5	0.00880	160.7	5
	2	520.34	0	0.000	0.0	17872.5	0.00880	165.8	2
	3	272	0	0.000	0.0	17873.5	0.00880	163.3	0.5
	4	26.83	0.25	0.000	0.0	17874.5	0.00880	160.7	1
CS9	1	296.9	0	19.994	0.1	13011	0.01230	162.9	1
	2	634.27	0	20.594	0.1	13012	0.01230	165.5	5
	3	296.96	0	21.212	0.1	13013	0.01230	162.9	2
	4	634.37	0	21.848	0.1	13014	0.01230	165.5	0.5
CS10	1	500.99	0.32	218.300	22.0	10283.5	0.01542	162.5	5
	2	752.75	0.01	295.548	0.0	10284.5	0.01542	164.9	0.5
	3	176.11	0	202.048	0.0	10285.5	0.01542	160.2	1
	4	541.99	0	124.776	0.0	10286.5	0.01542	162.5	2
CS11	1	35.74	99999999	0.000	0.0	16681	0.00950	159.3	0.5
	2	146.26	99999999	100.065	9.9	16682	0.00950	161.1	2
	3	321.75	99999999	8.992	5.0	16683	0.00950	162.9	5
	4	192.43	99999999	361.805	0.0	16684	0.00950	161.1	1
CS14	1	24.75	99999999	540.321	11.6	12649.5	0.01226	156.1	1
	2	314.55	99999999	3.776	10.9	12650.5	0.01226	158.7	2
	3	167.88	99999999	800.949	5.1	12651.5	0.01226	157.4	0.5
	4	339.5	99999999	335.469	3.3	12652.5	0.01226	158.7	5

CritShr (Pa)	EroCoeff (cm^3/Ns)	Layer 1					
		Thck (m)	WetP (m)	FriAng (degrees)	Coh (kPa)	SUW (kN/m^3)	Unsat (degrees)
0.00024948	6.3311	2	3.087	30	3	18	15
0.00024948	6.3311	2	3.087	30	3	18	15
0.00024948	6.3311	2	3.087	30	3	18	15
0.00024948	6.3311	2	3.087	30	3	18	15
0.00024948	6.3311	2	3.087	30	3	18	15
0.00024948	6.3311	2.1	5.143	30	3	18	15
0.00024948	6.3311	2.1	5.143	30	3	18	15
0.00024948	6.3311	2.1	5.143	30	3	18	15
0.00024948	6.3311	2.1	5.143	30	3	18	15
0.00024948	6.3311	2.1	5.143	30	3	18	15
0.00024948	6.3311	2.3	4.503	30	3	18	15
0.00024948	6.3311	3.3	4.503	30	3	18	15
0.00024948	6.3311	4.3	4.503	30	3	18	15
0.00024948	6.3311	5.3	4.503	30	3	18	15
0.00024948	6.3311	1	2.417	30	3	18	15
0.00024948	6.3311	1	2.417	30	3	18	15
0.00024948	6.3311	1	2.417	30	3	18	15
0.00024948	6.3311	1	2.417	30	3	18	15
0.00024948	6.3311	0.7	13.318	25	10	18	15
0.00024948	6.3311	0.7	13.318	25	10	18	15
0.00024948	6.3311	0.7	13.318	25	10	18	15
0.00024948	6.3311	0.7	13.318	25	10	18	15
0.00024948	6.3311	2.3	8.228	20	15	18	15
0.00024948	6.3311	2.3	8.228	20	15	18	15
0.00024948	6.3311	2.3	8.228	20	15	18	15
0.00024948	6.3311	2.3	8.228	20	15	18	15
0.00024948	6.3311	2.8	5.131	30	3	18	15
0.00024948	6.3311	2.8	5.131	30	3	18	15
0.00024948	6.3311	2.8	5.131	30	3	18	15
0.00024948	6.3311	2.8	5.131	30	3	18	15
0.00024948	6.3311	1.2	5.727	36	0	18	15
0.00024948	6.3311	1.2	5.727	36	0	18	15
0.00024948	6.3311	1.2	5.727	36	0	18	15
0.00024948	6.3311	1.2	5.727	36	0	18	15
0.00024948	6.3311	1.2	2.332	25	10	18	15
0.00024948	6.3311	1.2	2.332	25	10	18	15
0.00024948	6.3311	1.2	2.332	25	10	18	15
0.00024948	6.3311	1.2	2.332	25	10	18	15
0.00024948	6.3311	2	28.371	20	15	18	15
0.00024948	6.3311	2	28.371	20	15	18	15
0.00024948	6.3311	2	28.371	20	15	18	15
0.00024948	6.3311	2	28.371	20	15	18	15

Layer 2						Layer 3		
Thck (m)	WetP (m)	FriAng (degrees)	Coh (kPa)	SUW (kN/m^3)	Unsat (degrees)	Thck (m)	WetP (m)	FriAng (degrees)
5	13.426	25	10	18	15	2.2	4.561	25
5	13.426	25	10	18	15	2.2	4.561	25
5	13.426	25	10	18	15	2.2	4.561	25
5	13.426	25	10	18	15	2.2	4.561	25
5	13.426	25	10	18	15	2.2	4.561	25
2	9.024	20	15	18	15	2	3.124	20
2	9.024	20	15	18	15	2	2.759	20
2	9.024	20	15	18	15	2	2.759	20
2	9.024	20	15	18	15	2	2.759	20
2	9.024	20	15	18	15	2	2.759	20
1.4	2.608	20	15	18	15	5.1	6.401	25
1.4	2.625	20	15	18	15	5.1	6.375	25
1.4	2.625	20	15	18	15	5.1	6.375	25
1.4	2.625	20	15	18	15	5.1	6.375	25
1.3	3.087	27	0	18	15	3.2	23.420	20
1.3	3.087	27	0	18	15	3.2	23.420	20
1.3	3.087	27	0	18	15	3.2	23.420	20
1.3	3.087	27	0	18	15	3.2	23.420	20
3.5	28.218	20	15	18	15	3.6	10.534	20
3.5	28.218	20	15	18	15	3.6	10.534	20
3.5	28.218	20	15	18	15	3.6	10.534	20
3.5	28.218	20	15	18	15	3.6	10.534	20
2.1	6.075	20	15	18	15	2.2	5.646	20
2.1	6.075	20	15	18	15	2.2	5.646	20
2.1	6.075	20	15	18	15	2.2	5.646	20
2.1	6.075	20	15	18	15	2.2	5.646	20
3	5.492	36	0	18	15	0.9	2.193	25
3	5.492	36	0	18	15	0.9	2.193	25
3	5.492	36	0	18	15	0.9	2.193	25
3	5.492	36	0	18	15	0.9	2.193	25
1.1	5.266	36	0	18	15	0.7	3.032	25
1.1	5.266	36	0	18	15	0.7	3.032	25
1.1	5.266	36	0	18	15	0.7	3.032	25
1.1	5.266	36	0	18	15	0.7	3.032	25
1.4	2.280	20	15	18	15	1.5	2.421	20
1.4	2.280	20	15	18	15	1.5	2.421	20
1.4	2.280	20	15	18	15	1.5	2.421	20
1.4	2.280	20	15	18	15	1.5	2.421	20
1	7.071	20	15	18	15	1	6.675	25
1	7.071	20	15	18	15	1	6.675	25
1	7.071	20	15	18	15	1	6.675	25
1	7.071	20	15	18	15	1	6.675	25

Layer 3			Layer 4					
Coh (kPa)	SUW (kN/m³)	Unsat (degrees)	Thck (m)	WetP (m)	FriAng (degrees)	Coh (kPa)	SUW (kN/m³)	Unsat (degrees)
10	18	15	3	32.903	27	0	18	15
10	18	15	3	32.903	27	0	18	15
10	18	15	3	29.415	27	0	18	15
10	18	15	3	29.415	27	0	18	15
10	18	15	3	29.415	27	0	18	15
15	18	15	2.1	10.708	20	15	18	15
15	18	15	2.1	9.856	20	15	18	15
15	18	15	2.1	4.669	20	15	18	15
15	18	15	2.1	4.669	20	15	18	15
15	18	15	2.1	4.669	20	15	18	15
10	18	15	3.1	8.954	25	10	18	15
10	18	15	3.1	8.954	25	10	18	15
10	18	15	3.1	8.954	25	10	18	15
10	18	15	3.1	8.954	25	10	18	15
15	18	15	3	66.867	20	15	18	15
15	18	15	3	22.788	20	15	18	15
15	18	15	3	19.729	20	15	18	15
15	18	15	3	16.466	20	15	18	15
15	18	15	3.7	29.731	20	15	18	15
15	18	15	3.7	29.731	20	15	18	15
15	18	15	3.7	9.823	20	15	18	15
15	18	15	3.7	8.273	20	15	18	15
15	18	15	2.1	61.236	20	15	18	15
15	18	15	2.1	61.236	20	15	18	15
15	18	15	2.1	61.236	20	15	18	15
15	18	15	2.1	61.236	20	15	18	15
10	18	15	2.1	9.339	25	10	18	15
10	18	15	2.1	9.339	25	10	18	15
10	18	15	2.1	9.339	25	10	18	15
10	18	15	2.1	9.339	25	10	18	15
10	18	15	0.5	1.649	25	10	18	15
10	18	15	0.5	1.649	25	10	18	15
10	18	15	0.5	1.649	25	10	18	15
10	18	15	0.5	1.649	25	10	18	15
15	18	15	1.4	3.311	20	15	18	15
15	18	15	1.4	3.311	20	15	18	15
15	18	15	1.4	3.311	20	15	18	15
15	18	15	1.4	3.311	20	15	18	15
10	18	15	1	6.280	25	10	18	15
10	18	15	1	6.280	25	10	18	15
10	18	15	1	6.280	25	10	18	15
10	18	15	1	6.280	25	10	18	15

Layer 5

Thck (m)	WetP (m)	FriAng (degrees)	Coh (kPa)	SUW (kN/m^3)	Unsat (degrees)
2.5	18.630	25	10	18	15
2.5	18.630	25	10	18	15
2.5	22.110	25	10	18	15
2.5	22.110	25	10	18	15
2.5	22.110	25	10	18	15
2.3	38.768	36	0	18	15
2.3	40.136	36	0	18	15
2.3	45.588	36	0	18	15
2.3	45.588	36	0	18	15
2.3	45.588	36	0	18	15
1.6	60.021	36	0	18	15
1.6	60.021	36	0	18	15
1.6	60.021	36	0	18	15
1.6	60.021	36	0	18	15
1.8	35.046	36	0	18	15
1.8	79.230	36	0	18	15
1.8	82.320	36	0	18	15
1.8	85.629	36	0	18	15
2.8	24.461	36	0	18	15
2.8	24.461	36	0	18	15
2.8	44.788	36	0	18	15
2.8	46.484	36	0	18	15
4.6	55.061	36	0	18	15
4.6	55.061	36	0	18	15
4.6	55.061	36	0	18	15
4.6	55.061	36	0	18	15
0.9	150.003	36	0	18	15
0.9	150.003	36	0	18	15
0.9	150.003	36	0	18	15
0.9	150.003	36	0	18	15
1.7	111.713	36	0	18	15
1.7	111.713	36	0	18	15
1.7	111.713	36	0	18	15
1.7	111.713	36	0	18	15
1.9	48.337	36	0	18	15
1.9	48.337	36	0	18	15
1.9	48.337	36	0	18	15
1.9	48.337	36	0	18	15
3.4	121.947	36	0	18	15
3.4	121.947	36	0	18	15
3.4	121.947	36	0	18	15
3.4	121.947	36	0	18	15

APPENDIX B.

AVERAGE APPLIED SHEAR STRESS RESULTS

Model #	p	F Value	α	R^2	MSE	Cp	intercept
1	1	29.63	<0.0001	0.4256	30872.000	107.894	-6632.0279
2	2	31.50	<0.0001	0.6177	21075.000	61.105	-9076.5441
3	3	24.80	<0.0001	0.6619	19125.000	51.863	-9689.4035
4	4	20.49	<0.0001	0.6890	18071.000	46.996	-11039
5	5	18.64	<0.0001	0.7213	16640.000	40.772	-12062
6	6	16.00	<0.0001	0.7328	16410.000	39.859	-12056
7	6	20.06	<0.0001	0.7747	13826.000	29.212	-146622
8	6	23.63	<0.0001	0.8020	12162.000	22.293	-17973
9	6	24.02	<0.0001	0.8046	12002.000	21.629	-19254
10	6	29.12	<0.0001	0.8331	10252.000	14.392	-20899
11	6	29.57	<0.0001	0.8352	10119.000	13.845	-20444
12	7	29.75	<0.0001	0.8597	8872.963	9.642	-20850
13	8	27.84	<0.0001	0.8710	8406.421	8.775	-21034
14	8	28.58	<0.0001	0.8739	8216.937	8.036	-20903
15	9	27.22	<0.0001	0.8845	7762.041	7.346	-20253
16	10	24.84	<0.0001	0.8891	7692.749	8.175	-20145
17	11	22.24	<0.0001	0.8908	7827.359	9.743	-20471
18	12	19.74	<0.0001	0.8909	8085.960	11.705	-20474
19	12	19.78	<0.0001	0.8911	8070.179	11.650	-20490
20	12	19.79	<0.0001	0.8912	8066.554	11.638	-20667
21	12	19.81	<0.0001	0.8913	8060.482	11.617	-20328
22	12	19.81	<0.0001	0.8913	8058.231	11.609	-19135
23	13	17.66	<0.0001	0.8913	8344.630	13.605	-18949
24	13	19.54	<0.0001	0.9007	7622.500	11.216	156644
25	14	17.80	<0.0001	0.9023	7782.030	12.824	164301
26	14	18.37	<0.0001	0.9050	7565.531	12.134	-1039975
27	14	18.37	<0.0001	0.9050	7563.890	12.128	-1005163
28	15	16.53	<0.0001	0.9051	7846.444	14.103	-1033904
29	15	16.54	<0.0001	0.9051	7844.847	14.098	-1272320
30	15	16.54	<0.0001	0.9051	7844.627	14.097	-1247430
31	16	14.91	<0.0001	0.9051	8157.985	16.096	-1216257
32	16	14.97	<0.0001	0.9055	8129.315	16.011	-867236
33	16	14.97	<0.0001	0.9055	8125.568	16.000	-306931
34	17	13.53	<0.0001	0.9055	8464.110	18.000	-299998

Model #	reachL	reachS	FlowELE	Tflow	CritShr	EroCoeff
1			42.53078			
2			56.27132			
3			60.47775			
4			68.45583			
5			75.6083			
6			74.11869			
7			87.11205			
8			91.09094			
9		12115	90.61941			
10		17653	97.17799			
11	-0.02703		98.29644			
12	-0.03355		102.40487			
13	-0.03514		102.09537			
14	-0.03382		104.46849			
15	-0.03637		105.29025			
16	-0.03608		104.75808	8.74951		
17	-0.03807		105.2365	9.22679		
18	-0.03793		105.32936	9.10155		
19	-0.03352		105.32505	9.3517		
20	-0.02617		105.35554	9.32741		
21	-0.02517		105.36193	9.37552		
22	-0.08609	-34493	105.37468	9.35215		
23	-0.08892	-36311	105.37961	9.37045		
24	-2.51768	-1120461	111.12439	11.27299		
25	-2.32986	-1214588	11.70675	11.25604		
26	11.1065	15456032	111.60373	11.82049		
27	10.64.350	14986515	111.62984	11.84192		
28	11.04173	15360915	111.62962	11.8142		
29	12.69061	19716019	111.44056	11.82796		
30	12.28633	19401788	111.49116	11.82601		
31	11.71415	18892854	111.56047	11.82439		
32	12.03223	13256589	111.55255	11.91882		
33	12.36575	7084118	111.45131	11.94396		
34	12.36432	6942556	111.45783	11.94284		

Model #	Layer 1					
	Thck	WetP	FriAng	Coh	SUW	Unsat
1						
2						
3	-53.72422					
4	-54.84708	6.45807				
5	-49.33318	7.70941				
6	-62.20447	7.0473				
7	-86.3343			18.49272		
8	-52.60703		76.42965	89.67284		
9			118.7473	133.36519		
10			140.80478	170.62094		
11			139.3329	172.48383		
12			137.48221	178.33089		
13			145.96105	189.19545		
14			132.49697	177.72539		
15			112.97999	163.0222		
16			111.95497	161.92101		
17			118.94692	167.26823		
18	6.79667		118.96555	168.68741		
19	10.446	2.74745	119.04082	166.28643		
20	7.75311	8.27138	116.60258	160.86082		
21	15.03107	9.56746	100.58942	150.38017		
22	10.8089		98.40392	167.02424		
23	13.28876		92.46007	164.08545		
24			-5927.26097	-2796.35505		
25			-6412.81353	-3331.6463		
26			-2882.8017			
27	7.22453		-2940.02906			
28	7.26768		-2862.73031			
29	7.28868		-4293.37565			
30	7.28048		-4428.80115			
31	7.46779		-4516.54536			
32	17.76411	2896.4337				
33	23.35651	5282.94488		-3689.67925		
34	22.94786	5192.08714		-3730.72193		

Model #	Layer 2					
	Thck	WetP	FriAng	Coh	SUW	Unsat
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15		5.59738				
16		5.69373				
17		6.32518		4.26676		
18		6.57152		2.56429		
19		5.08386				
20			7.27575			
21	-15.90987		16.40808			
22	-17.2105		18.96871			
23	-22.95095		22.43308			
24	-7614.20442		3380.26039			
25	-7749.02014		3462.29			
26	41270		-125.95287			
27	39806					
28	41029		-126.85937			
29	65461		-588.82737			
30	63151			398.12093		
31	63040	-39.30413		289.9071		
32	51144	-2088.84429		-1103.96897		
33	34291	-3190.13124		-2985.88242		
34	32222	-3109.81477		-2958.07664		

Model #	Layer 3					
	Thck	WetP	FriAng	Coh	SUW	Unsat
1						
2						
3						
4						
5						
6			12.49758			
7			29.66562			
8			25.17357			
9			7.18506			
10						
11						
12	-37.72061					
13	-40.68878					
14						
15						
16						
17						
18						
19						
20						
21						
22						
23						
24		-708.79352				
25		-575.19676	316.00277			
26		-677.93419	23900			
27		-715.4434	23147			
28		-670.49829	23758			
29	9879.35254	-667.18563	29377			
30	93.1407372	-668.48791	28421			
31	9898.50328	-670.62815	27973			
32	14677	683.2685	14443			
33	22442	-680.10067				
34	21175	-680.40387				

Model #	Layer 4					
	Thck	WetP	FriAng	Coh	SUW	Unsat
1						
2						
3						
4						
5	-58.29592					
6	-54.84331					
7	-58.1126					
8						
9						
10						
11						
12						
13		1.84061				
14	-72.24463	2.62402				
15	-120.17173	3.73269				
16	-121.90124	3.84013				
17	-127.35367	4.95083				
18	-129.05138	4.64539				
19	-120.35475	4.32148				
20	-86.93205	3.22786				
21	-79.03611	2.09779				
22	-76.71522	2.08845				
23	-73.90725	1.68283				
24	6049.80731	-351.15169				
25	5828.5613	-373.71555				
26	-35408	-421.17658		19731		
27	-34101	-418.92144		19166		
28	-35208	-418.65314		19612		
29	-66796	-413.7867		26479		
30	-64375	-415.24374		25835		
31	-64603	-417.25243		25553		
32	-44200	-422.03813		16013		
33	-29905	-419.26059		5043.65571		
34	-27357	-419.5012		4714.9329		

Model #	Layer 5					
	Thck	WetP	FriAng	Coh	SUW	Unsat
1						
2		2.70721				
3		3.06988				
4		2.93099				
5		2.35462				
6		2.06432				
7		2.60273				
8		3.11064				
9		3.54882				
10	-83.68942	3.48525				
11	-86.65138	3.22945				
12	-110.05367	2.67547				
13	-143.05444	2.69056				
14	-155.62583	2.61031				
15	-187.35632	2.51299				
16	-189.53483	2.48644				
17	-207.78948	2.89859				
18	-206.22219	2.67915				
19	-202.78	2.19711				
20	-186.76632	1.10993				
21	-174.47282					
22	-168.56852					
23	-164.06092	-0.40975				
24	4631.79504	-350.59379				
25	5088.74894	-372.88741				
26	3094.43878	-420.75206				
27	3099.46873	-418.51874				
28	3073.86349	-418.2429				
29		-413.50579				
30		-414.92907				
31		-416.89071				
32		421.66355				
33		-418.92691				
34	335.9234	-419.16552				

APPENDIX C.

FACTOR OF SAFETY RESULTS

Model #	p	F Value	α	R²	MSE	Cp	Intercept
1	1	14.19	0.0005	0.2619	1.83E+15	0.000	112990213
2	2	20.36	<0.0001	0.5108	1.24E+15	0.000	211542790
3	3	22.96	<0.0001	0.6444	9.27E+14	0.000	300320113
4	4	31.97	<0.0001	0.7756	6.01E+14	0.000	430424539
5	5	49.58	<0.0001	0.8732	3.49E+14	0.000	407418912
6	6	112.51	<0.0001	0.9507	1.39E+14	0.000	495278862
7	7	183.44	<0.0001	0.9742	7.51E+13	0.000	492479474
8	7	220.85	<0.0001	0.9785	6.27E+13	0.000	133004173
9	7	249.46	<0.0001	0.9809	5.56E+13	0.000	-7523612
10	8	500.40	<0.0001	0.9918	2.45E+13	0.000	-112686155
11	8	16441.4	<0.0001	0.9997	7.53E+11	0.000	-158286873
12	8	20802.5	<0.0001	0.9998	5.95E+11	0.000	-123095852
13	9	18589.9	<0.0001	0.9998	5.92E+11	0.000	-102765147
14	9	1.4E+08	<0.0001	1.0000	7.98E+07	0.000	368159855
15	9	∞	<0.0001	1.0000	0.00E+00	0.000	-1450276415
16	10	∞	<0.0001	1.0000	0.00E+00	0.000	-1450288695
17	11	∞	<0.0001	1.0000	0.00E+00	0.000	-1450291550
18	12	∞	<0.0001	1.0000	0.00E+00	0.000	-1450291546
19	13	∞	<0.0001	1.0000	0.00E+00	0.000	-1450291909
20	14	∞	<0.0001	1.0000	0.00E+00	0.000	-1450296788
21	15	∞	<0.0001	1.0000	0.00E+00	0.000	-1450297044
22	16	∞	<0.0001	1.0000	0.00E+00	0.000	-1450297068
23	17	∞	<0.0001	1.0000	0.00E+00	0.000	-1450298307

Model #	reachL	reachS	FlowELE	Tflow	CritShr	EroCoeff
1						
2						
3	-5245.52346					
4	-8468.49137					
5	8233.83523					
6	-7150.7769					
7	-6210.8696					
8	-6682.64803					
9		4902737508				
10		3563237055				
11		424726054				
12						
13						
14						
15						
16	0.09871					
17	0.1216					
18	0.1216					
19	0.12454		-0.01092			
20	0.12461		-0.01119			
21	0.12534		-0.01166	0.00749		
22	0.12563		-0.01045	0.00775		
23	0.12982		-0.00613	0.00973		

Model #	Layer 1					
	Thck	WetP	FriAng	Coh	SUW	Unsat
1	-26653279					
2	-28048315					
3	-30874001					
4	-26772226					
5	-23120107					
6	-23866300					
7	-17888533					
8	-14972824					
9	-14991100					
10	-9465471		3159168			
11			5059981			
12			5201065			
13			5120999			
14			18486974	15372667		
15			65950127	55473822		
16			65950509	55474110		
17			65950597	55474176		
18	-0.12017		65950597	55474176		
19	-0.12238		65950609	55474185		
20	-0.07557		65950736	55474292		
21	-0.07369		65950743	55474298		
22	-0.07363		65950743	55474298		
23	-0.07314		65950777	55474327		

Model #	Layer 2					
	Thck	WetP	FriAng	Coh	SUW	Unsat
1						
2			-3944082			
3			-4837874			
4			-7214627			
5			-756287			
6			-9543751			
7			-10055505			
8				10954892		
9				11201336		
10				12636164		
11	24010951			14311523		
12	20966282			14206245		
13	18231912	107293		14059569		
14	-132578637	3453742		7983876		
15	-386449445		45451030	52610881		
16	-386453301		45451671	52611470		
17	-386454198		45451820	52611607		
18	-386454198		45451821	52611607		
19	-386454313		45451840	552611624		
20	-586454995	-9.24603	45451962	52611744		
21	-386455047	-9.56052	45451971	52611753		
22	-386455065	-9.52254	45451974	52611756		
23	-386455380	-10.66966	45452030	52611805		

Model	Layer 3					
#	Thck	WetP	FriAng	Coh	SUW	Unsat
1						
2						
3						
4	-18948910					
5	-36305445					
6	-41396575					
7	-46724969	1887571				
8	-52887754	6751961				
9	-53204059	6545691				
10	-60550694	8407196				
11	-47471668	13518581				
12	-48291396	13460379				
13	-49258601	13212941				
14	-120807716					
15	-314113650					
16	-314116081					
17	-31411647					
18	-314116647					
19	-314116719					
20	-314117238					
21	-314117274					
22	-314117284					
23	-314117492	-0.79965				

Model #	Layer 4					
	Thck	WetP	FriAng	Coh	SUW	Unsat
1						
2						
3						
4						
5	27274320					
6	29810387					
7	29641575					
8	28876940					
9	26429032					
10	30020614					
11	-12738462					
12	-9302743					
13	-6217857					
14	201250215					
15	640447065					
16	640452809					
17	640454144					
18	640454144					
19	640454314					
20	640455495					
21	640455577					
22	640455602	-0.06589				
23	640456090	-0.36964				

Model #	Layer 5					
	Thck	WetP	FriAng	Coh	SUW	Unsat
1						
2						
3						
4						
5						
6	-19228843					
7	-22412239					
8	-27779769					
9	-30776568					
10	-27010039					
11	-34183132					
12	-34005062		-882271			
13	-33789200		-1313016			
14	-22606411		-24546423			
15	-2050835		-61998012			
16	-2050562		-61998649			
17	-2050499	-0.00664	-61998797			
18	-2050499	-0.00664	-61998797			
19	-2050491	-0.00669	-61998816			
20	-2050436	-0.00669	-61998917			
21	-2050432	-0.00704	-61998925			
22	-2050430	-0.07237	-61998928			
23	-2050403	-0.37431	-6199897			

APPENDIX D.

MAXIMUM LATERAL RETREAT

Model #	p	F Value	α	R^2	MSE	Cp	Intercept
1	1	13.05	0.0008	0.2460	41503.000	39.586	66.74605
2	2	16.56	<0.0001	0.4593	30524.000	19.635	-43.01585
3	3	14.46	<0.0001	0.5330	27059.000	14.055	4.23537
4	4	14.90	<0.0001	0.6170	22790.000	7.410	-46.47851
5	4	16.19	<0.0001	0.6364	21637.000	5.414	148.36726
6	4	18.13	<0.0001	0.6622	20103.000	2.762	-411.8471
7	5	14.74	<0.0001	0.6719	20067.000	3.762	-325.35717
8	6	12.99	<0.0001	0.6901	19494.000	3.888	-413.83099
9	6	13.50	<0.0001	0.6982	18985.000	3.054	-345.21412
10	6	14.21	<0.0001	0.7089	18310.000	1.951	519.67754
11	7	12.15	<0.0001	0.7144	18491.000	3.383	462.78502
12	7	12.16	<0.0001	0.7146	18479.000	3.363	279.55001
13	8	11.10	<0.0001	0.7290	18078.000	3.881	95.84303
14	8	11.59	<0.0001	0.7375	17515.000	3.012	1114.42973
15	9	10.14	<0.0001	0.7405	17857.000	4.706	1438.6259
16	9	10.56	<0.0001	0.7481	17329.000	3.917	11916
17	10	9.30	<0.0001	0.7500	17756.000	5.725	13912
18	10	9.51	<0.0001	0.7541	17465.000	5.304	6847.0675
19	11	8.48	<0.0001	0.7576	17853.000	7.030	5896.69792
20	11	8.65	<0.0001	0.7603	17590.000	6.663	-641733
21	12	7.68	<0.0001	0.7607	18168.000	8.624	-632485
22	13	6.86	<0.0001	0.7610	18792.000	10.591	-636598
23	14	6.14	<0.0001	0.7611	19483.000	12.585	-637475
24	15	5.52	<0.0001	0.7611	20229.000	14.581	-679827
25	15	5.70	<0.0001	0.7667	19757.000	14.007	108692
26	15	5.70	<0.0001	0.7667	19756.000	14.006	745090
27	16	5.14	0.0001	0.7668	20541.000	16.000	766264
28	16	5.14	0.0001	0.7668	20541.000	16.000	637265
29	17	4.64	0.0003	0.7668	21397.000	18.000	633076
30	17	4.64	0.0003	0.7668	21397.000	18.000	255919
31	17	4.64	0.0003	0.7668	21397.000	18.000	-8367402
32	17	4.64	0.0003	0.7668	21397.000	18.000	-260535399

Model #	reachL	reachS	FlowELE	Tflow	CritShr	EroCoeff
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12	0.03203					
13	0.04537					
14		-43509				
15		-64030				
16		-412166				
17		-453061				
18		-489971				
19	0.07235	-639775				
20	15.71176	16616367				
21	15.8383	16362003	2.6359			
22	15.60527	16469878	2.68338			
23	15.625	16492529	2.74181	-1.02907		
24	17.06079	17909983	2.65302	-0.9799		
25	13.34867	-5652861		-2.6153		
26	133.3317			-2.6332		
27	13.40346		-1.0752	-2.63788		
28	13.46682	2339774	-1.10923	-2.63904		
29	13.4703	2383334	-1.11299	-2.63935		
30	13.47029	10451593	-1.11299	-2.63935		
31	13.47031	113801216	-1.11299	-2.63936		
32	13.47157	325771705	-1.11286	-2.63935		

Model #	Layer 1					
	Thck	WetP	FriAng	Coh	SUW	Unsat
1						
2		14.33644				
3		13.36683				
4		10.79269				
5						
6						
7						
8				7.79845		
9				13.78427		
10				16.8228		
11	21.83266			16.82665		
12	40.36099					
13	74.77941					
14	90.90985					
15	117.59239			-11.23102		
16	118.97155			-247.25629		
17	121.76897		-26.7035	-297.12445		
18	122.35463		-309.00075	-592.48352		
19	118.76897		-649.4107	-993.03417		
20	118.82999		7301.4739	4988.8385		
21	119.4164		7193.18907	4908.17882		
22	119.26894		7237.73154	4923.4979		
23	119.24191		7247.63831	4931.56909		
24	118.88739		7504.63003	5040.81626		
25	90.30831		-29744	-33780		
26	92.57792	-1180.90437	-22949	-25495		
27	91.53306	-1318.17859	-23486	-25939		
28	92.4561	-785.38996	-20720	-2588		
29	92.55592	-17.8507342	-20591	-22471		
30	92.55582	-3784.05239	-13971	-12901		
31	92.55397	6013.15272	-84840	-8671.42331		
32	92.54256	26106	-230197	-354709		

Model #	Layer 2					
	Thck	WetP	FriAng	Coh	SUW	Unsat
1						
2						
3						
4		9.60254				
5	-94.55046	22.6753				
6	-130.75428	27.48514				
7	-116.59756	26.04989				
8	-93.68419	22.96345				
9	-16.57707	26.83416				
10	-160.47257	29.76419				
11	-163.41609	31.14897				
12	-201.92935	34.92412				
13	-277.61675	41.47404				
14	-277.51618	37.43781				
15	-319.05943	39.37639				
16	-361.43345		-92.39789			
17	-411.9467		-98.6819			
18	-917.15002		367.49352	423.87651		
19	-1561.06501		773.74648	823.88814		
20	22014			4203.46378		
21	21676			4144.38247		
22	21815			4180.5466		
23	21845			4185.06598		
24	22883	77.38763		4450.72703		
25	-76965	6289.13619		-339.29231		
26	-58474	5854.40074		-979.04151		
27	-59830	6042.53954		-951.77459		
28	-52293	5862.91519				
29	-51997	5843.54905	-32.09075			
30	-31836	5843.56799	1214.10097	4000.79048		
31	-137658	5843.93643	230353	187082		
32	5846.16741	700315	562580	35320		

Model #	Layer 3					
	Thck	WetP	FriAng	Coh	SUW	Unsat
1		19.07919				
2		19.09459				
3		22.5659				
4		22.40251				
5		16.89081				
6		16.44578	24.82199			
7		17.92345	20.86099			
8		20.30732	21.75513			
9		17.14336	24.07871			
10		15.97587				
11		16.87071				
12		27.13767				
13		30.20643		41.33077		
14		39.28115		70.97972		
15		49.97895		95.33455		
16		20.3136		80.7916		
17		260.32094		95.31171		
18		408.94907		438.94014		
19		628.97214		769.3309		
20	-18817	619.68134		-19617		
21	-18544	619.30065		-19327		
22	-18687	633.29422		-19470		
23	-18710	633.06823		-1995		
24	-20078	618.27336		-20893		
25	15796	1076.3967		24887		
26	4158.45892	1083.83886		12447		
27	3920.58031	1100.15154		12476		
28	-807.27809	1102.38417		7412.02184		
29	-898.88678	1102.66748		7284.5758		
30	-17222	1102.66737		-9044.53635		
31		1102.66942		147768		
32		1102.65531		469387		

Model #	Layer 4					
	Thck	WetP	FriAng	Coh	SUW	Unsat
1						
2						
3		-3.49444				
4		-4.00338				
5		-2.52959				
6						
7		-1.49361				
8		-2.73379				
9						
10				-18.16766		
11				-17.64731		
12				-34.01826		
13				-66.28474		
14				-87.13684		
15				-113.95389		
16				-340.76049		
17				-378.21442		
18				-585.18856		
19				-906.53962		
20				14433		
21				14211		
22		0.5548		14302		
23		0.51473		14322		
24		0.61871		15313		
25		241.58105		-27671		
26		246.3332		-17403		
27		255.84346		-17640		
28		257.3353		-13459		
29		257.51538		-13350		
30		257.51529				
31		257.51606				
32		257.507				

Model #	Layer 5					
	Thck	WetP	FriAng	Coh	SUW	Unsat
1						
2						
3						
4						
5						
6						
7						
8						
9	-71.48526					
10	-95.00097					
11	-95.77292					
12	-74.61515					
13	-115.55814					
14	-124.074					
15	-119.46719					
16	65.36682					
17	85.37062					
18						
19						
20						
21						
22						
23						
24						
25		239.4398				
26		244.15626				
27		253.59971				
28		255.07663				
29		255.25515				
30		255.25506				
31	-110712	255.25583				
32	-337779	255.24677				

APPENDIX E.

ERODED AREA – TOTAL RESULTS

Model #	p	F Value	α	R^2	MSE	Cp	Intercept
1	1	6.24	0.02	0.1349	125.050	73.252	1.08906
2	2	5.10	0.11	0.2074	117.508	65.929	-2.22252
3	3	4.20	0.01	0.2492	114.239	62.552	1.71669
4	3	4.81	0.01	0.2754	110.260	59.189	3.84946
5	4	7.26	0.00	0.4397	87.556	40.053	17.77369
6	5	7.16	<0.0001	0.4986	80.525	34.476	35.42435
7	6	7.50	<0.0001	0.5626	72.262	28.252	42.29978
8	7	10.12	<0.0001	0.6758	55.141	15.698	68.07279
9	8	9.41	<0.0001	0.6951	53.416	15.206	68.36874
10	9	8.17	<0.0001	0.6968	54.790	16.995	20.63477
11	9	8.36	<0.0001	0.7017	53.906	16.366	-879.20162
12	9	8.39	<0.0001	0.7023	53.800	16.291	-841.36931
13	10	7.36	<0.0001	0.7037	55.274	18.110	-834.74039
14	10	7.51	<0.0001	0.7078	54.492	17.571	-803.78588
15	11	6.62	<0.0001	0.7083	56.220	19.513	-855.97761
16	11	6.67	<0.0001	0.7098	55.940	19.326	-1312.63223
17	11	6.67	<0.0001	0.7098	55.933	19.321	-1044.04301
18	11	8.23	<0.0001	0.7510	47.985	14.017	59300
19	12	7.54	<0.0001	0.7574	48.378	15.204	59871
20	13	6.79	<0.0001	0.7591	49.746	16.980	61312
21	14	6.08	<0.0001	0.7592	51.568	18.968	61446
22	15	5.47	<0.0001	0.7593	53.533	20.957	61934
23	15	7.53	<0.0001	0.8129	41.619	14.067	450080
24	16	6.81	<0.0001	0.8134	43.163	16.000	455146
25	17	6.15	<0.0001	0.8134	44.961	18.000	459529
26	17	6.15	<0.0001	0.8134	44.961	18.000	-39520

Model #	reachL	reachS	FlowELE	Tflow	CritShr	EroCoeff
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17		-474.42238				
18	-3.09185	-1762227				
19	-3.10978	-1772521				
20	-3.18931	-1817800	0.29019			
21	-3.19614	-1821685	0.29458	-0.06633		
22	-3.19066	-1824884	0.29102	-0.06843		
23	-2.73427	-5055282	0.96626			
24	-2.71238	-5091037	0.96572	0.15649		
25	-2.71362	-5113617	0.96437	0.15544		
26	-2.71362	1743915	0.96438	0.15544		

Model #	Layer 1					
	Thck	WetP	FriAng	Coh	SUW	Unsat
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11			3.31891			
12			3.30056			
13	-0.79384		3.20574			
14	-2.98686		3.85009			
15	-2.54385	-0.16925	4.00953			
16	-2.62605	-1.24412		-4.74752		
17	-2.57008	-0.34687		-4.05282		
18		-491.84783		937.84584		
19	-2.68661	-494.10869		943.67799		
20	-2.63705	-506.83241		967.95585		
21	-2.63773	-507.91903		970.05174		
22	-2.68279	-509.08854		964.54867		
23	-5.92015	-1240.61071		-985.43907		
24	-5.95985	-1248.18368		-1017.11033		
25	-6.02683	-1268.39084		-1027.35939		
26	-6.02685	-1301.07403	-1699.9548	1535.81539		

Model #	Layer 2					
	Thck	WetP	FriAng	Coh	SUW	Unsat
1		0.59583				
2		0.57085				
3	-2.50377	0.85117				
4	-4.14816	1.00092				
5	-6.6657	1.4539				
6	-10.83483	1.87216				
7	-12.53881	1.98476		0.64951		
8	-17.09871	2.6092		1.48958		
9	-16.62278	2.54121		1.84928		
10	-15.78345	2.35222	1.27079	3.22058		
11		-1.10926	21.88946	27.17153		
12		-1.00598	20.92578	26.19135		
13		-1.04912	20.87034	26.0374		
14		-0.92525	19.02953	25.06421		
15		-0.95302	20.27349	26.63932		
16		-2.0462	36.95716	45.03606		
17		-18.4946	30.02785	36.12837		
18		17.38627	57.54072	-45.94439		
19		17.7077	51.64768	-53.46852		
20		18.06396	54.1545	-53.58464		
21		18.10987	54.17801	-53.81626		
22		19.00918	43.47124	-60.83469		
23		569.23139	-7718.04797	-6567.24073		
24		576.73252	-7825.6054	-6656.98782		
25		590.25684	-7912.87498	-6720.37581		
26		590.26057	3179.86609	2885.06127		

Model #	Layer 3					
	Thck	WetP	FriAng	Coh	SUW	Unsat
1						
2		0.53167				
3		0.42938				
4						
5						
6						
7						
8	-4.78929					
9	-6.31318	0.46827				
10	-6.83419	1.06361				
11	-20.42915	11.79844				
12	-20.22404	11.41561				
13	-19.55086	11.26346				
14	-18.56836	10.60632				
15	-19.99903	11.35681				
16	-31.53802	19.36483				
17	-24.50835	15.6109				
18	-99.52289	23.00877				
19	-93.63814	19.66062				
20	-93.90003	20.70483				
21	-97.02127	20.68723				
22	-103.83696	22.71592		-12.23202		
23	-1614.16504	-52.93274		-6484.87103		
24	-1634.38566	-54.14867		-6574.23221		
25	-1702.66195	-53.92546		-6686.63528		
26	-4924.5559	-53.92551		-3339.82377		

Model #	Layer 4					
	Thck	WetP	FriAng	Coh	SUW	Unsat
1						
2						
3						
4		0.17474				
5		0.4313				
6		0.53139		-0.97166		
7		0.67607		-1.27142		
8		1.05376		-1.76426		
9		1.05453		-1.85451		
10		1.05145		-1.76244		
11		1.04278		0.25799		
12		1.04794				
13		1.03985				
14		1.19902				
15		1.21352				
16		1.17814				
17		1.04811				
18		0.87131				
19		0.87379				
20		0.87109				
21		0.86803				
22		0.86473				
23		-40.83883				
24		-41.41413				
25		-41.29948		23.20313		
26		-41.2995		3321.17317		

Model	Layer 5					
#	Thck	WetP	FriAng	Coh	SUW	Unsat
1						
2						
3						
4						
5	-7.09672					
6	-8.21162					
7	-12.3702					
8	-20.79174					
9	-22.11958					
10	-22.58517					
11	-24.92271					
12	-24.6781			-0.43622		
13	-24.50942			-0.40132		
14	-25.53577	0.14593				
15	-25.87696	0.16084				
16	-26.32641	0.13358				
17	-24.21611					
18	197.81274					
19	200.24265					
20	205.37292					
21	205.90779					
22	213.57606					
23	5091.14929	-41.42071				
24	5157.37194	-41.99942				
25	5210.88685	-41.88551				
26		-41.88554				

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VITA

Annabell Leigh Ulary was born on August 19, 1989 in Austin, TX. She earned a Bachelor of Science in Civil Engineering degree from Texas Tech University in December of 2010. Annabell has also earned a Master of Science in Civil Engineering degree from Missouri University of Science and Technology in May 2013.